Hilroy

80 Pages
27.6 cm x 21.2 cm

Ruled 7 mm • Ligné 7 mm

EXERCISE BOOK
CAHIER D'EXERCICES

NAME/NOM Ecole Shanks

SUBJECT/SUJET ELEC 221

ASSEMBLED IN CANADA WITH IMPORTED MATERIALS
ASSEMBLÉ AU CANADA AVEC DES MATIÈRES IMPORTÉES

12107

ELEC 221 LEC 1

1/04/19
LEC

Signal → Contains information

System → Can generate or transform a signal

Continuous → Discrete

- Continuous-time signal $x(t)$ can be converted to a discrete-time signal $x[n]$ through sampling

Difference:
$t \in$ Real numbers
$n \in$ integers

sampling period

$$x[n] = x(nT_s) = x(t)\Big|_{t=nT_s}$$

Continuous Signals

1/08/19
LEC

Classifications

① Deterministic → represented by a formula vs. Stochastic → probabalistic

domain ② Continuous time (CT), $D \in \mathbb{R}$ vs. Discrete time (DT), $D \in \mathbb{Z}$

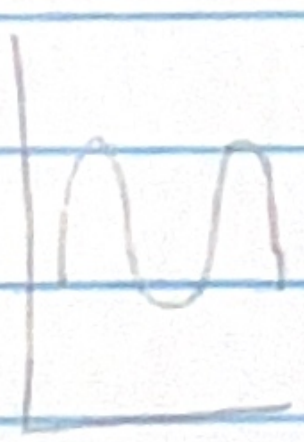
Range ③ Continuous amplitude vs. Discrete amplitude

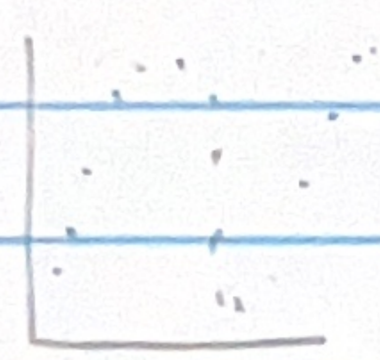
Analog → Continuous time/amplitude

Digital → Discrete time/Amplitude

ADC → Analog to Digital Converter

$x(t) \to$ Sampler $\xrightarrow{x_1}$ Quantizer $\xrightarrow{x_2}$ encoder $\xrightarrow{x_3}$





{5.3, ... 9.6}

1010

- Continuous sinosoidal

- Discrete samples
- DT
- CT amplitude

- 16 values
- DT
- DT amplitude

- Binary representation

Signal Operations

Differentiator

$$x(t) \rightarrow \boxed{\frac{d}{dt}} \rightarrow x'(t)$$

$$x(t) \rightarrow \boxed{S} \quad x'(t)$$
$$X(S) \rightarrow \qquad sX(s)$$

Integrator

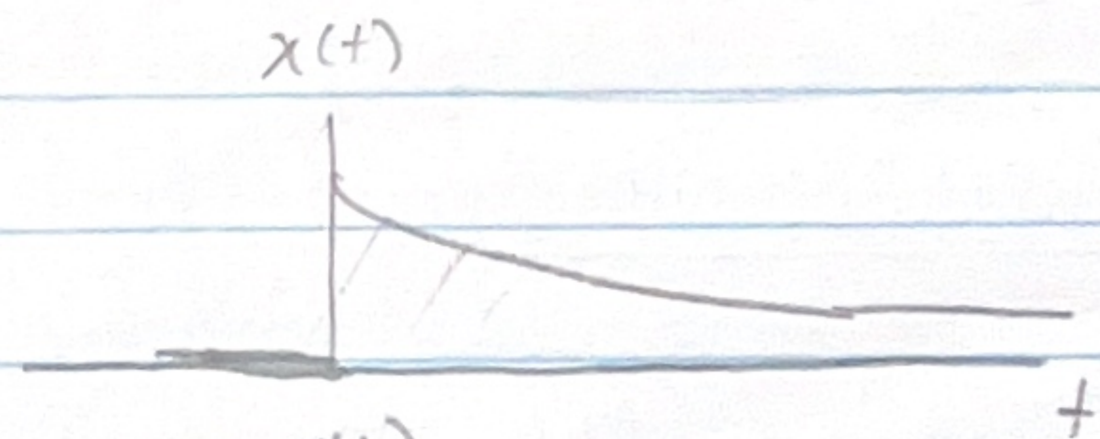
$$x(t) \rightarrow \boxed{\int} \rightarrow x_0 + \int_0^t x(t)\,dt$$

$$X(s) \rightarrow \boxed{\frac{1}{s}} \rightarrow \frac{X(s)}{s} + \frac{x_0}{s}$$

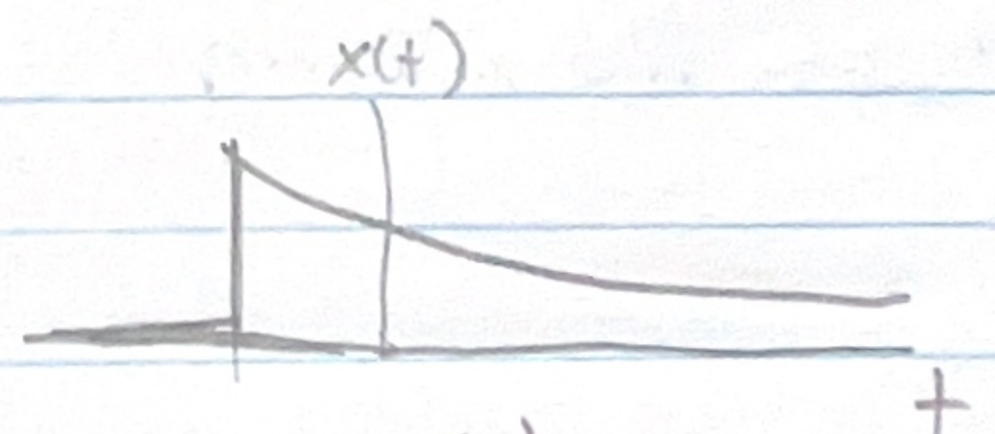
0/19
EC

Signal Classifications

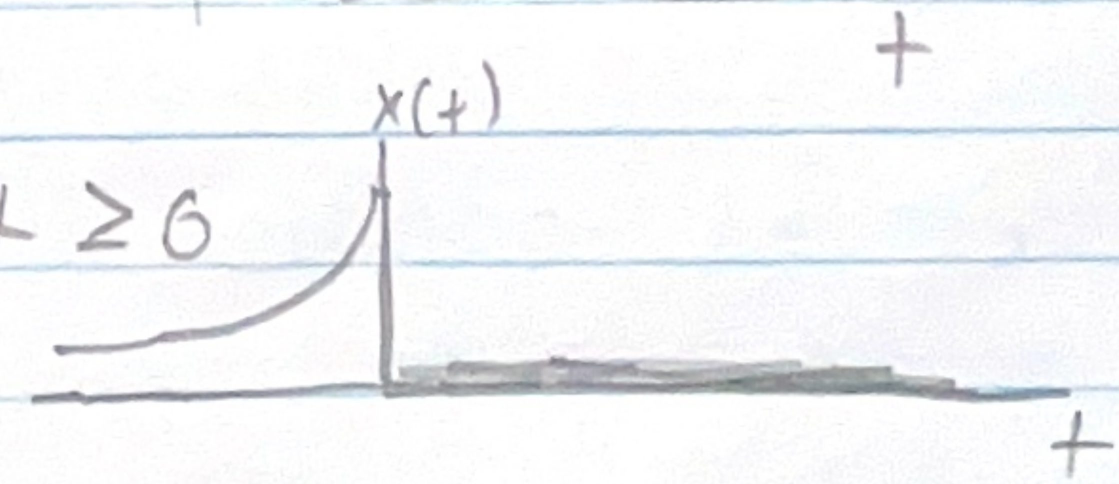
Causal $x(t) = 0$ For: $t < 0$



A Causal $x(t) \neq \exists t < 0$

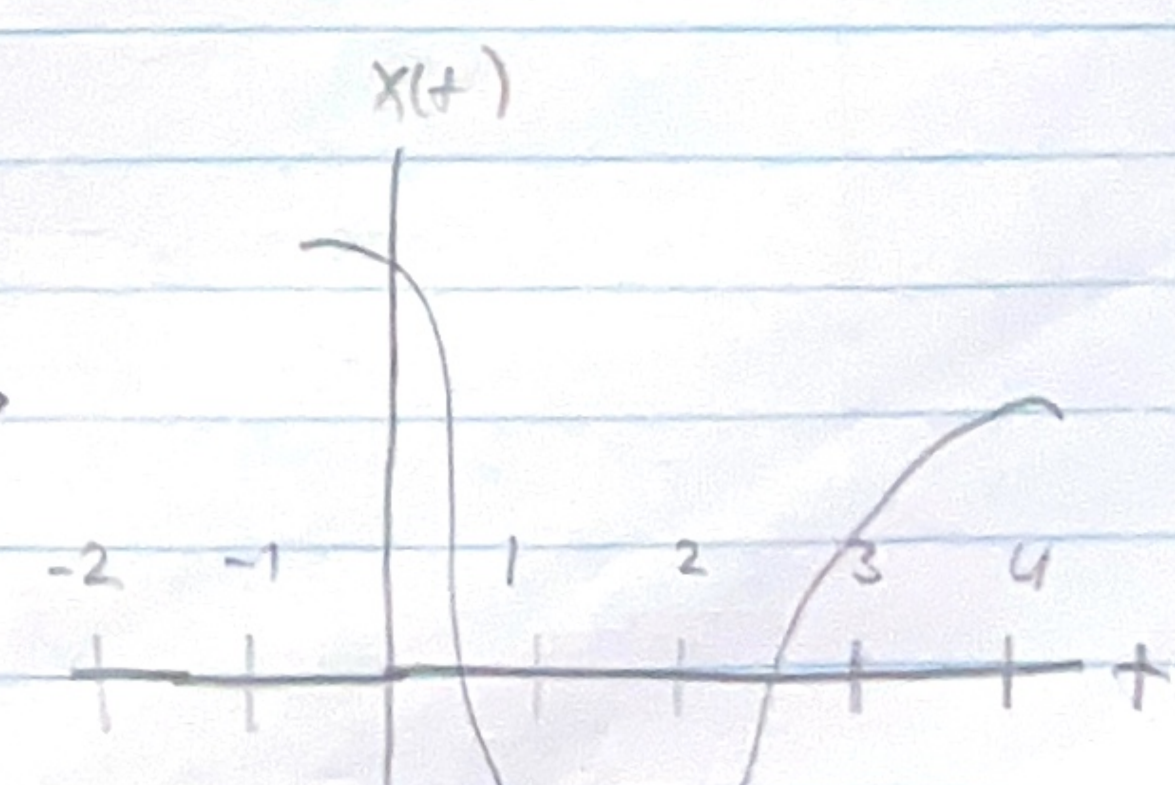


Anti Causal $x(t) = 0$ For: $t \geq 0$



Characterize $\rightarrow x(t) = \sqrt{2}\cos\left(\frac{\pi t}{2} + \frac{\pi}{4}\right)$

1. Deterministic
2. Continuous time
3. Continuous amplitude $\rightarrow$ analog
4. Shifted left 45°
5. Period of 4

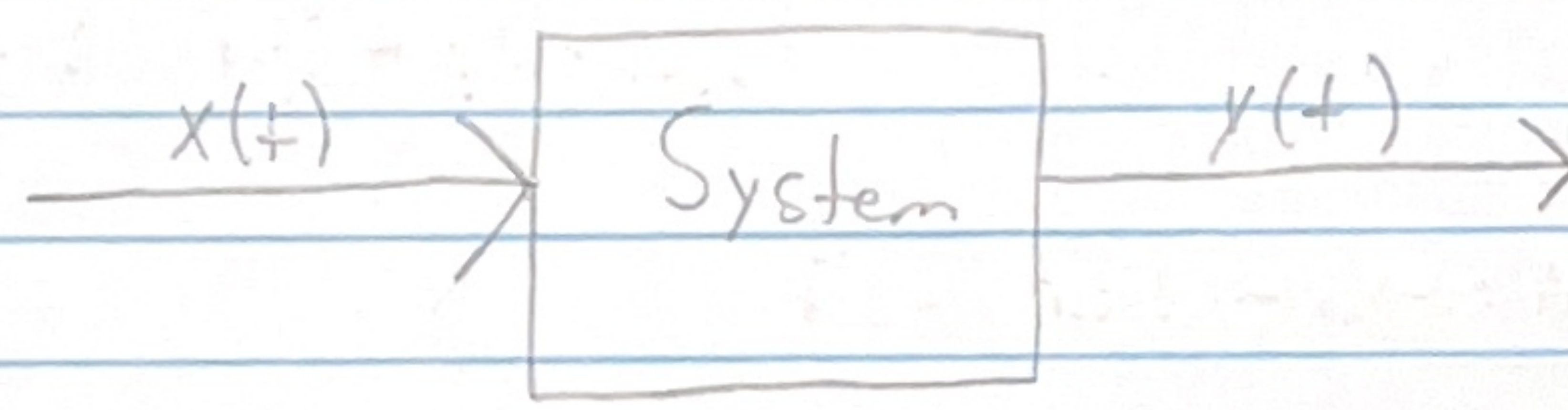


Even/odd Functions

$$x_E(t) = \frac{1}{2}[x(t) + x(-t)] \qquad x_O(t) = \frac{1}{2}[x(t) - x(-t)]$$

Continuous Time Systems

System => function that takes domain signal and outputs range signal



$y = S(x)$

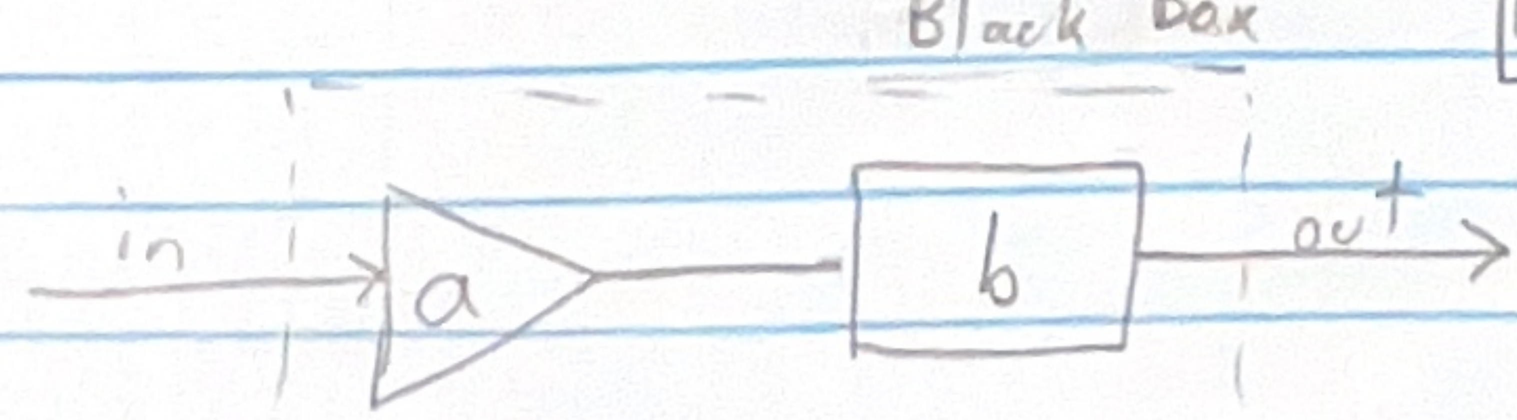
Classifications (Circuit elements)

memory vs. No memory

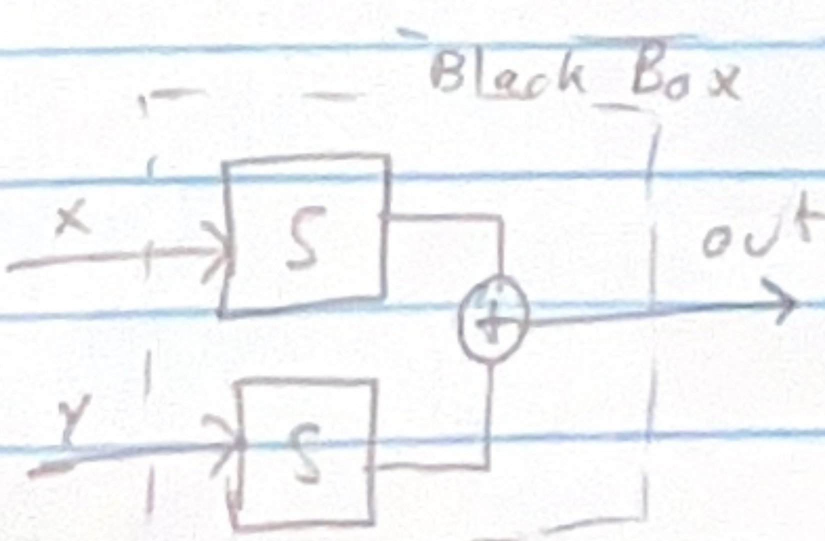
Distributed vs. Lumped (don't care inside the resistor)

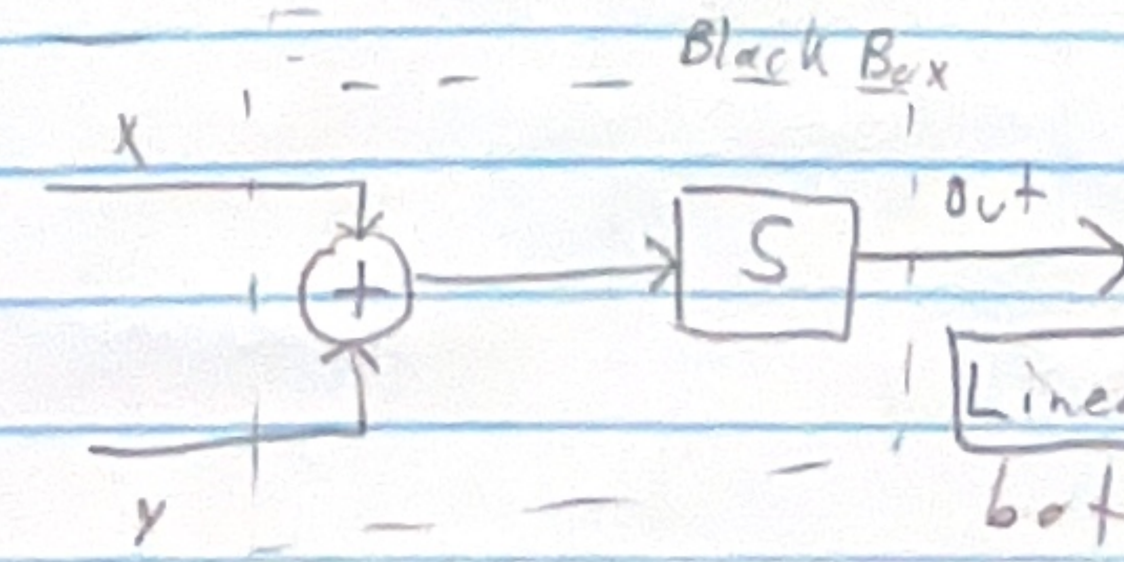
Passive vs Active

Linearity



Homogeneity → can't tell if you swap a and b inside the black box





Additivity → can't tell the difference

Linearity = satisfies both

Convolutions

1/17/19
LEC

① $Re(x) - jIm(x) \rightarrow Re(x-1) - jIm(x-1)$

② $(x-1) \longrightarrow Re(x-1) - jIm(x-1)$

Invariant

① $x(2t-1)$

② $x(2t-2)$

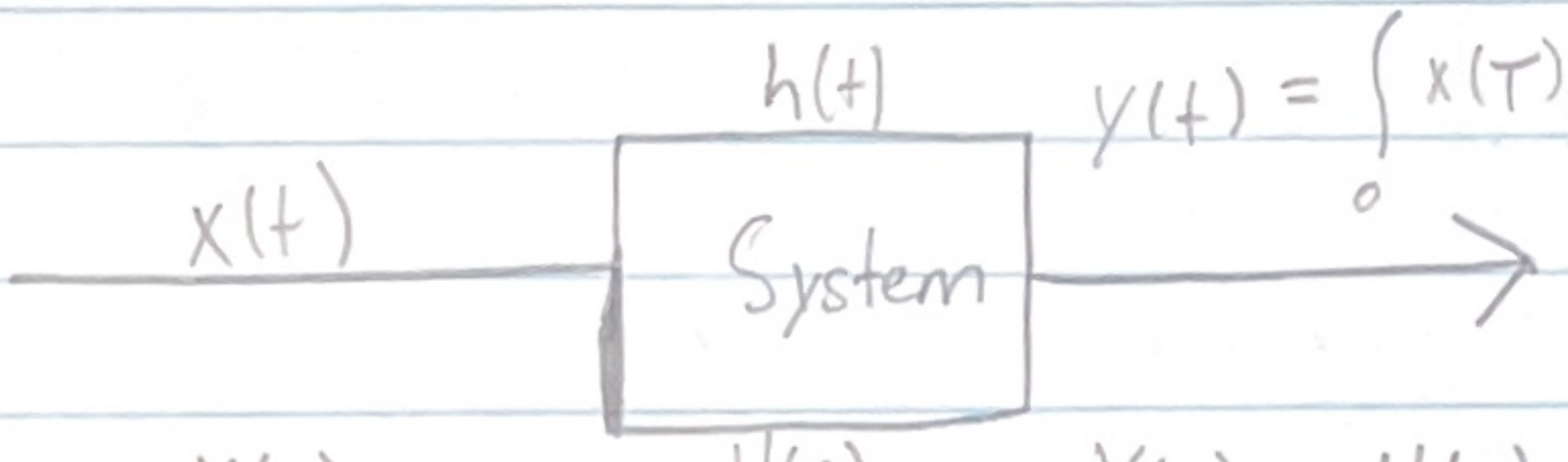
Not Invariant

Time Invariance

* If input shifted then fed through function is equal to inputs fed through function then outputs shifted. You have time Invariance

$$y(t \pm \tau) = S[x(t \pm \tau)]$$

Chapter 3 Laplace Transform



$$y(t) = \int_0^t x(\tau)h(t-\tau)\,d\tau = [x(t) * h(t)]$$

$$Y(s) = H(s)X(s)$$

① $f(t) = \delta(t) + 2u(t) - 3e^{-2t}$

$$F(s) = 1 + \frac{2}{s} - \frac{3}{s+2}$$ ROC: $Re(s) > 0$

Laplace Transforms

1/24/19
LEC

Types of Poles

① Simple

② Repeated

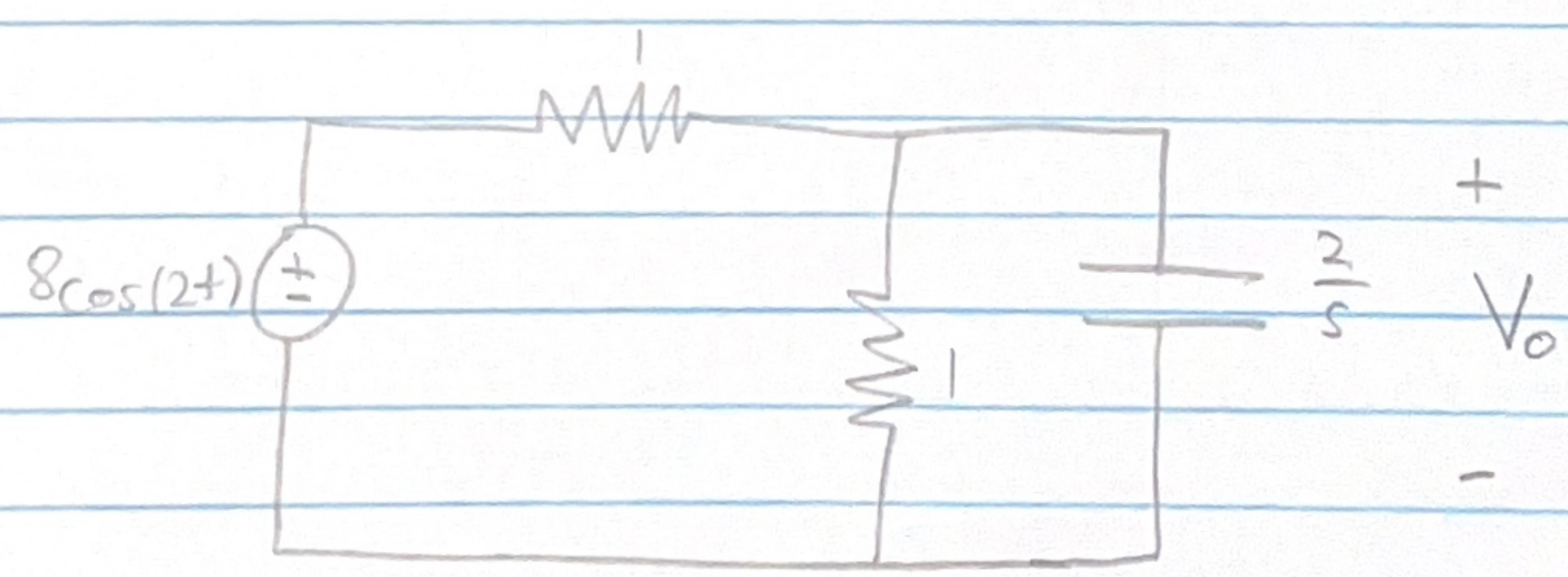
③ Complex

$$\frac{s^3+2s+6}{s(s+1)^2(s+3)}$$

$$\frac{A}{s}+\frac{B}{s+1}+\frac{C}{(s+1)^2}+\frac{D}{s+3}$$

$$A(s+1)^3(s+3)+B(s+1)^2(s)(s+3+C(s)(s+3)+D(s+1)^2 s$$

Transfer Functions



* $\boxed{\text{Impulse Response}}$ = Inverse Laplace transform of Transfer Function

* $\boxed{\text{Unit Step Response}}$ = $\frac{1}{s}$ · Transfer Function then Inverse Laplace Transform

Laplace Transform

$x(t) = u(t) - u(t-1)$

$\cancel{y(t) = }$

$x(t) = u(t) - u(t-1) \qquad X(s) = \frac{1}{s} - \frac{e^{-s}}{s}$

$y(t) = u(t) - u(t-1) - u(t-2) + u(t-3)$

$Y(s) = \frac{1}{s} - \frac{e^{-s}}{s} - \frac{e^{-2s}}{s} + \frac{e^{-3s}}{s}$

$Y(s) = H(s)\,X(s)$

$\therefore H(s) =$

$x(t) = x(t)$

$y(t) = x(t) - x(t-2)$

$x(t) = u(t) - u(t-0.35)$

$y(t) = u(t) - u(t-0.35) - u(t-2) - u(t-2-0.35)$

$x(t) = \delta(t)\,x(t)$

Fourier Series

SS 5.3 → Complex Exponential Representation

SS 5.6 → $X|_{k=0}$ gives avg value → DC value

SS 5.9 → $C_k = Re(X_k)$

$D_k = -Im(X_k)$

$[\cos(2\pi t)] \cdot [2\cos(2\pi t)]$

$= 2\cos^2(2\pi t)$

X_0 → DC Term

$s = jk\omega_0$ where,

$k\omega_0 = \omega_1, \omega_2, \ldots$

$\frac{\omega_0}{G.C.F}$ $\frac{T_0}{L.C.M}$

Frequency Response → Normal laplace transfer fcn $\frac{Y(s)}{X(s)}$ evaluated at $s = jk\omega_0$

$$\varepsilon_1 E_1 = \varepsilon_2 E_2$$

$$4.25 \langle 25, -5, -37 \rangle = 4.75 \langle x, y, z \rangle$$

$$4.25 \cdot 25 = 4.75x \rightarrow x = 22$$
$$4.25 \cdot (-5) = 4.75y \rightarrow y = -4.47$$
$$4.25 \cdot (-37) = 4.75z \rightarrow z = -33.10$$

$\boxed{\text{Fourier Transforms}}$

$$V = \frac{L}{\sigma S} I = IR$$

Known:

$L = 190$

$S = \pi(0.2E-3)^2$

$\sigma = 5\times10^6 \rho^2$

$$R = \frac{L}{\sigma S}$$

$$R = \frac{190}{\pi(0.2E-3)^2} \cdot \frac{1}{5\times10^6}$$

$$\int_0^{0.2E-3} \rho^2 \, d\rho = \frac{1}{3}(0.2E-3)^3$$

$L = 190$

$A = \pi(0.2E-3)^2$

$\sigma = 5\times10^6 \rho^2$

$$\frac{1}{3}(0.2E-3)^3$$

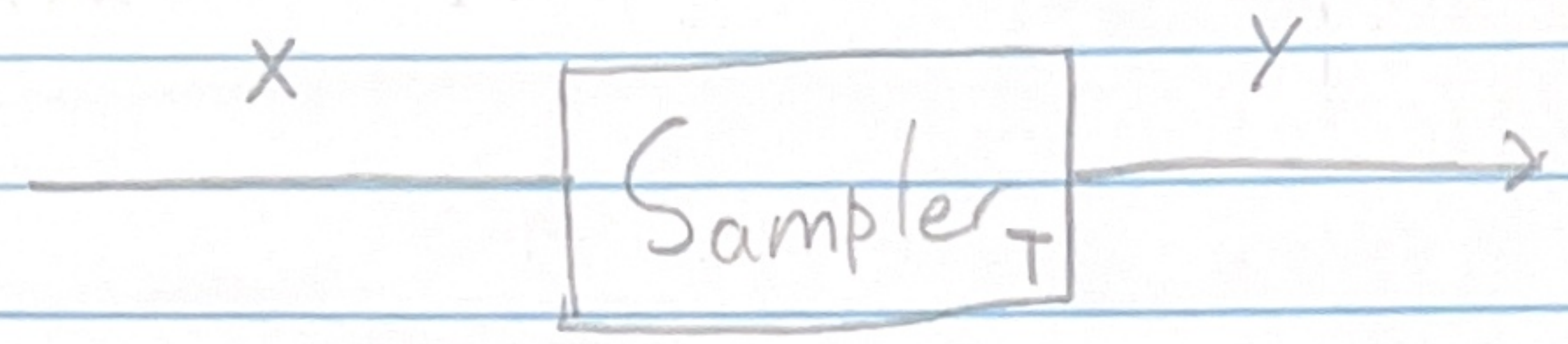
$$R = \frac{L}{\sigma A}$$

$$R = \frac{190}{\pi(0.2E-3)^2 \cdot 5\times10^6} \cdot \frac{1}{\sigma(\rho)}$$

Sampling

2/26/19
LEC

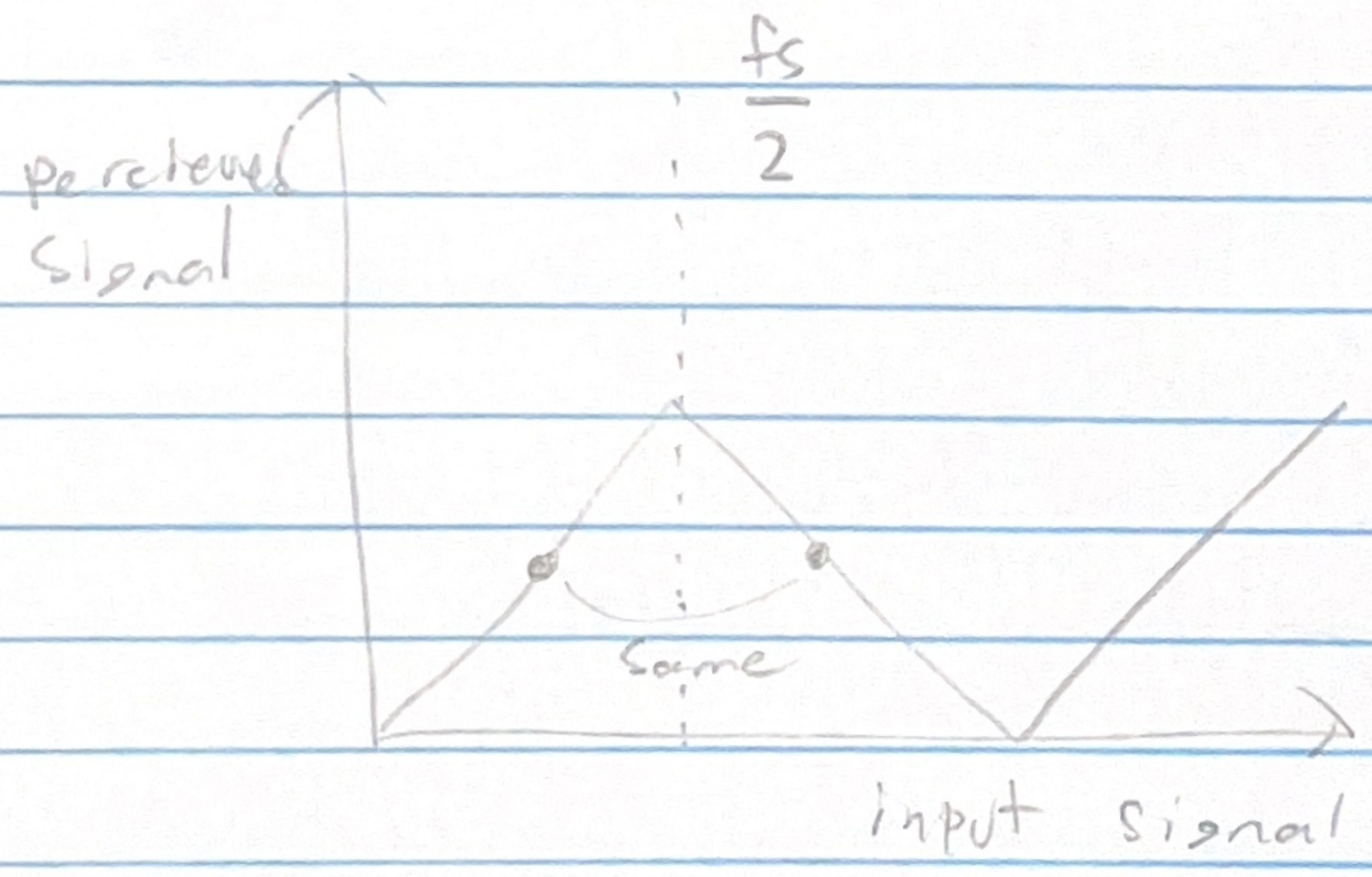
Uniform Sampling



$y = \text{Sampler}_T(x)$ $\qquad$ $y[n] = x(nT)\ ;\ n \in \mathbb{Z}$

Where:
T = Sampling interval
$f_s = 1/T$ = sampling frequency

SS 7.5



Sampling

$$x(t) \longrightarrow \boxed{\begin{array}{c} h(t) \\ \text{FILTER} \end{array}} \xrightarrow{\;y(t)\;}$$

$$\left.\begin{array}{l} |H(j\omega)| \\ \angle H(j\omega) \end{array}\right\} \text{freq response}$$

LPF
HPF
BPF
BRF (BSF)

<u>What type of filter?</u>

$$H(s) = \frac{1}{1+s}$$

$$H(j\omega) = \frac{1}{1+j\omega}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

$\therefore$ For $\omega \to 0$, $|H(j\omega)| \to 1$
For $\omega \to \infty$, $|H(j\omega)| \to 0$

LOW-PASS FILTER

What type of filter?

$$H(s) = \frac{s}{1+s}$$

$$H(j\omega) = \frac{j\omega}{1+j\omega}$$

$$|H(j\omega)| = \frac{\omega}{\sqrt{1+\omega^2}}$$

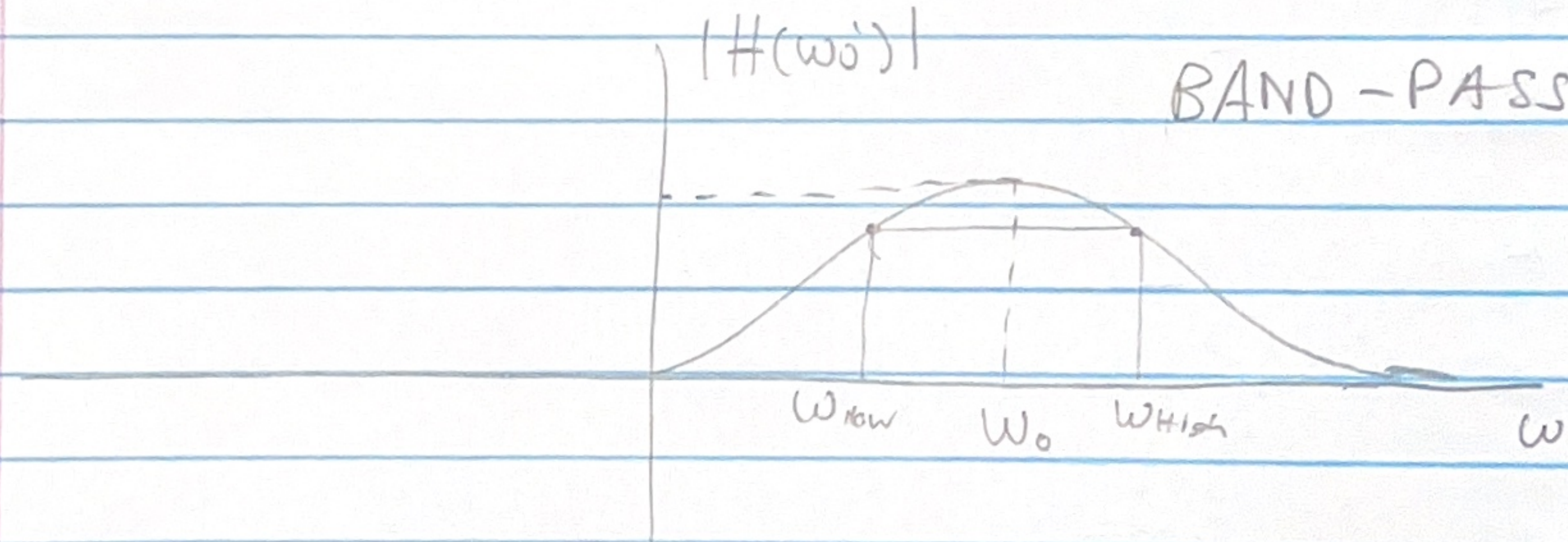
$\therefore$ For $\omega \to 0$, $|H(j\omega)| \to 0$
For $\omega \to \infty$, $|H(j\omega)| \to 1$

$\therefore$ HIGH-PASS FILTER

What type of Filter?

$$H(s) = \frac{ks}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

where: ξ = damping (zeta)
ω_0 = natural frequency



BAND-PASS

What type of filter?

$$H(s) = \frac{s^2 + \omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

BAND-STOP

$[w] = rad/s$ $\qquad w = 2\pi f$

$[\Omega] = rad/sample$ $\qquad [f] = \frac{Cycles}{sec} = Hz$

$[T] = sec/sample$

$wT = \Omega$

$\boxed{\text{Discrete Time Signals / Systems}}$

$$\frac{2\pi m}{N} = 1$$

$$\frac{m}{N} = \frac{1}{2\pi}$$

$\boxed{SS\ 8.8}$ $0.1w_0 = 0.1\pi$

$$\downarrow$$

$$2\pi \frac{m}{N} = 0.1\pi$$

$$\downarrow$$

$$\frac{m}{N} = \frac{1}{20} \rightarrow \therefore N = 20$$

$x(t) = \cos(2\pi(f_1 + \frac{fs}{2})t)$ <u>where:</u> fs = sampling rate

$\frac{fs}{2}$ = folding FREQ

$0 < f_1 < fs/2$

→ expect percieved frequency
$f_p = \frac{fs}{2} - f_1$ WHY?!

$$x_s[n] = \cos(2\pi(f_1 + \frac{fs}{2})nT_s)$$

Discrete time Signals

Basic DT signals

Unit step $\rightarrow$ $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$

Chronic Delta $\rightarrow$ $\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$

Recursive System

if $x[n] = \delta[n] \rightarrow$ Never goes to zero (IIR)

Discrete Time Systems

3/8/19
TUT

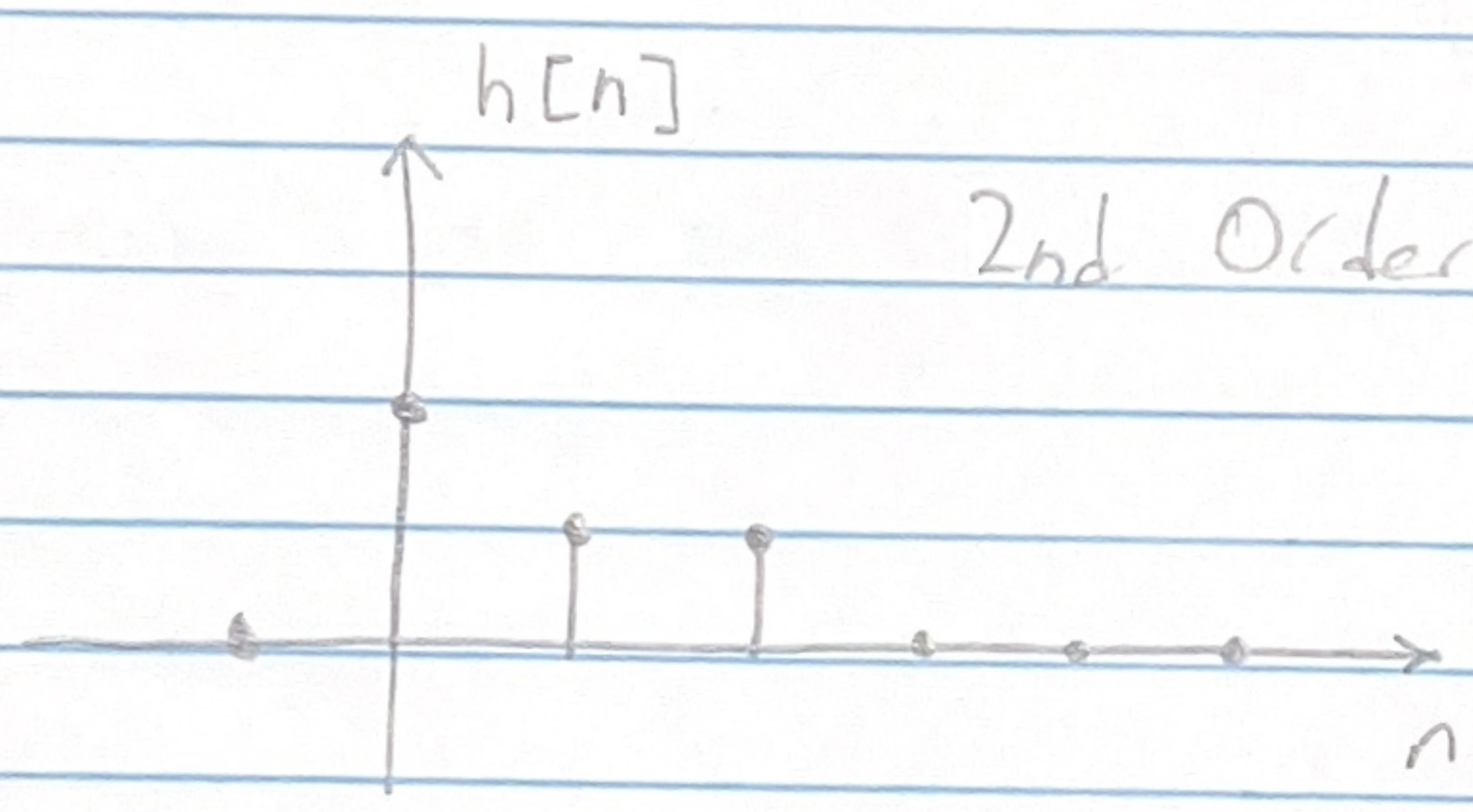
SS 8.23

System Order → max(N, M)

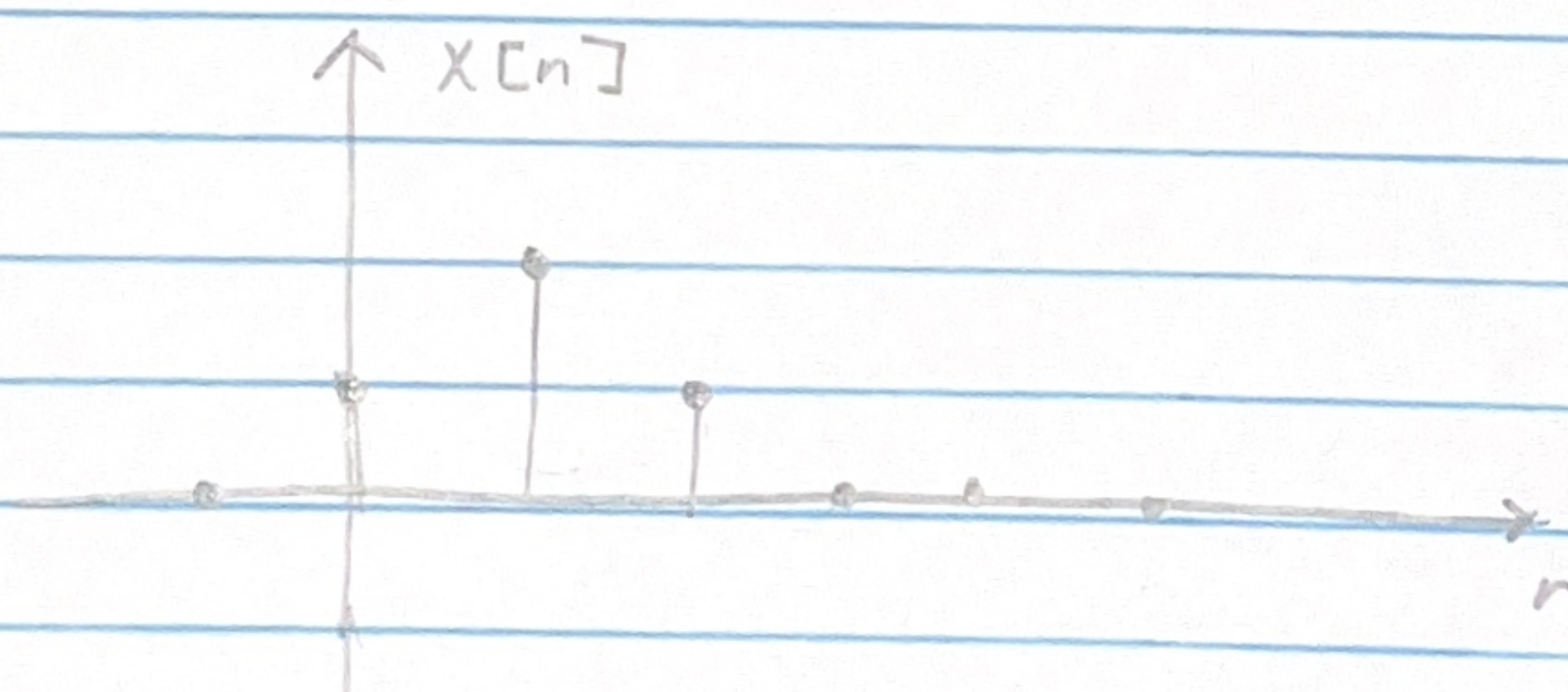
replace M-1, N-1 with M, N

SS 8.24

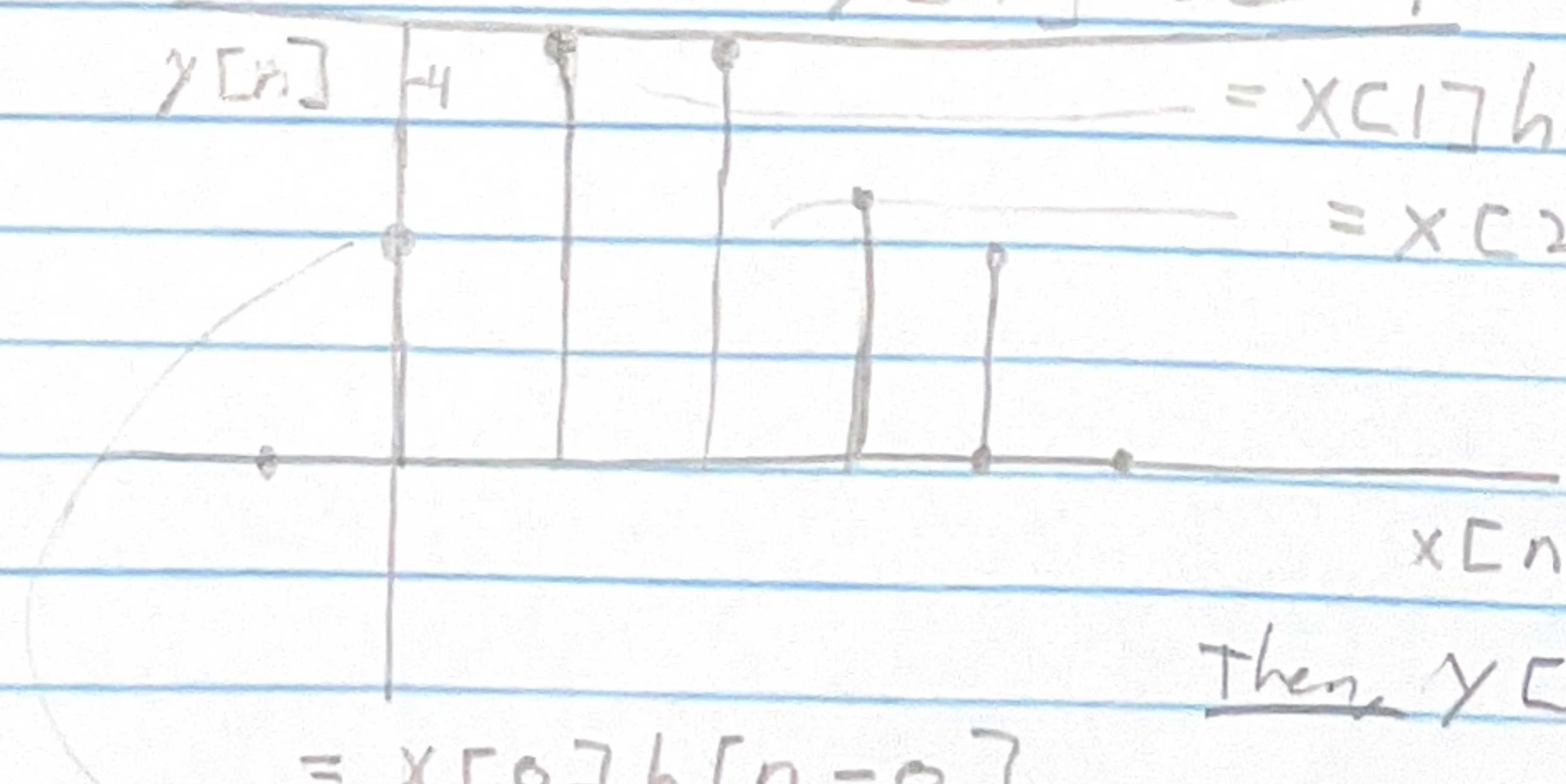
<u>Convolution Sum:</u>



2nd Order impulse response



<u>What will y[n] be?:</u>



$= x[1]h[n-1]$

$= x[2]h[n-2]$

$= x[0]h[n-0]$

<u>Then</u> y[n] = summing all these terms up

Convolution sum

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{m=-\infty}^{\infty} x[n-m]h[m]$$

scaling factors for time shifted function

Causal LTI DT system with causal input:

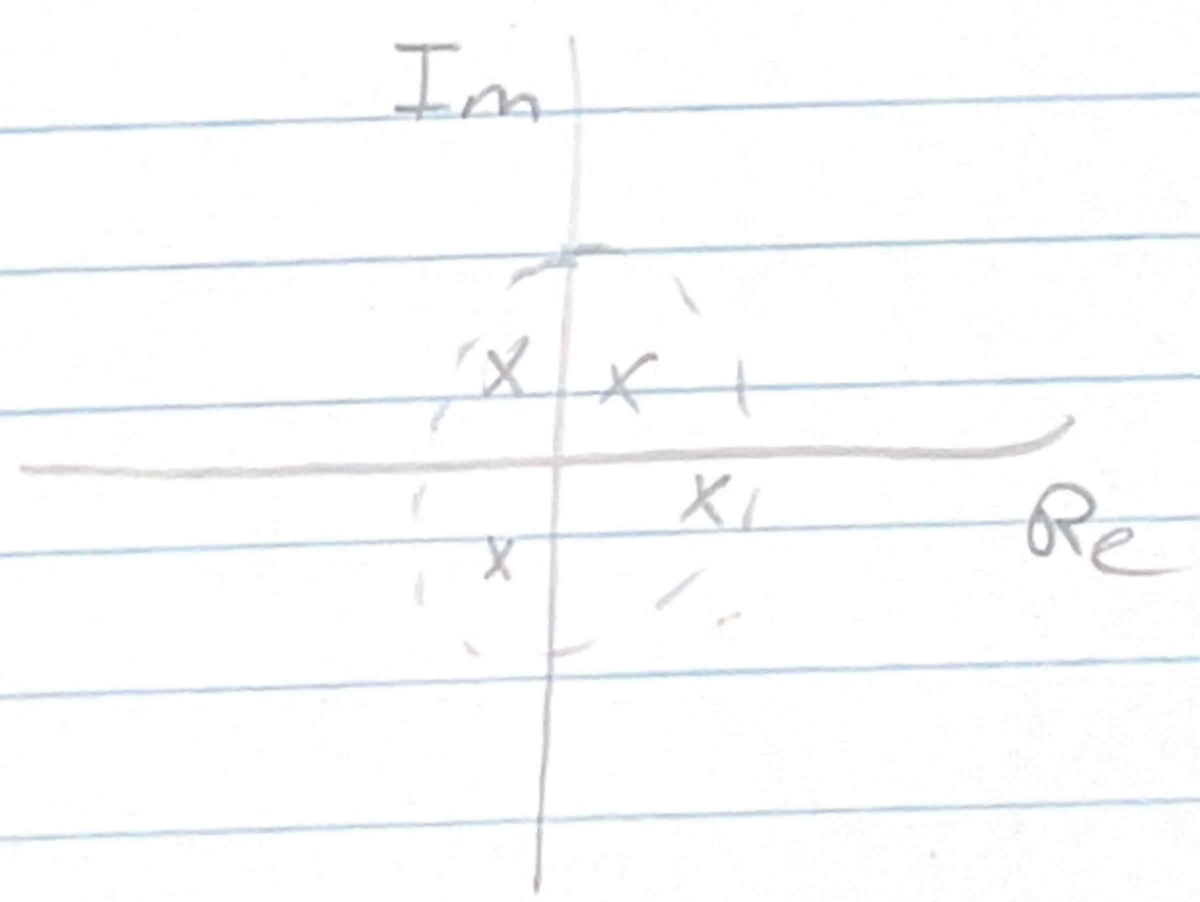
$$y[n] = \sum_{k=0}^{n} x[k]h[n-k], \quad n \geq 0$$

BIBO stable

$$\sum_{k} |h[k]| < \infty$$

Asymptotically stable

all poles of transfer function are within unit circle



Z - Transform

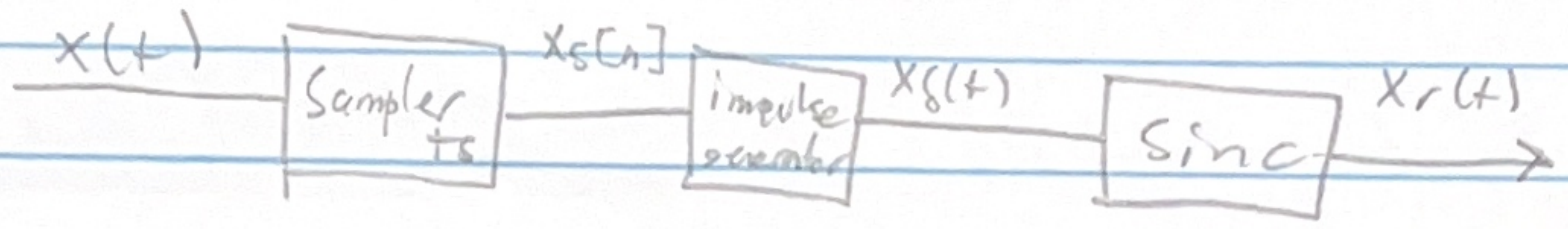
3/12/19
LEC

goes to n, if h[n] is causal

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

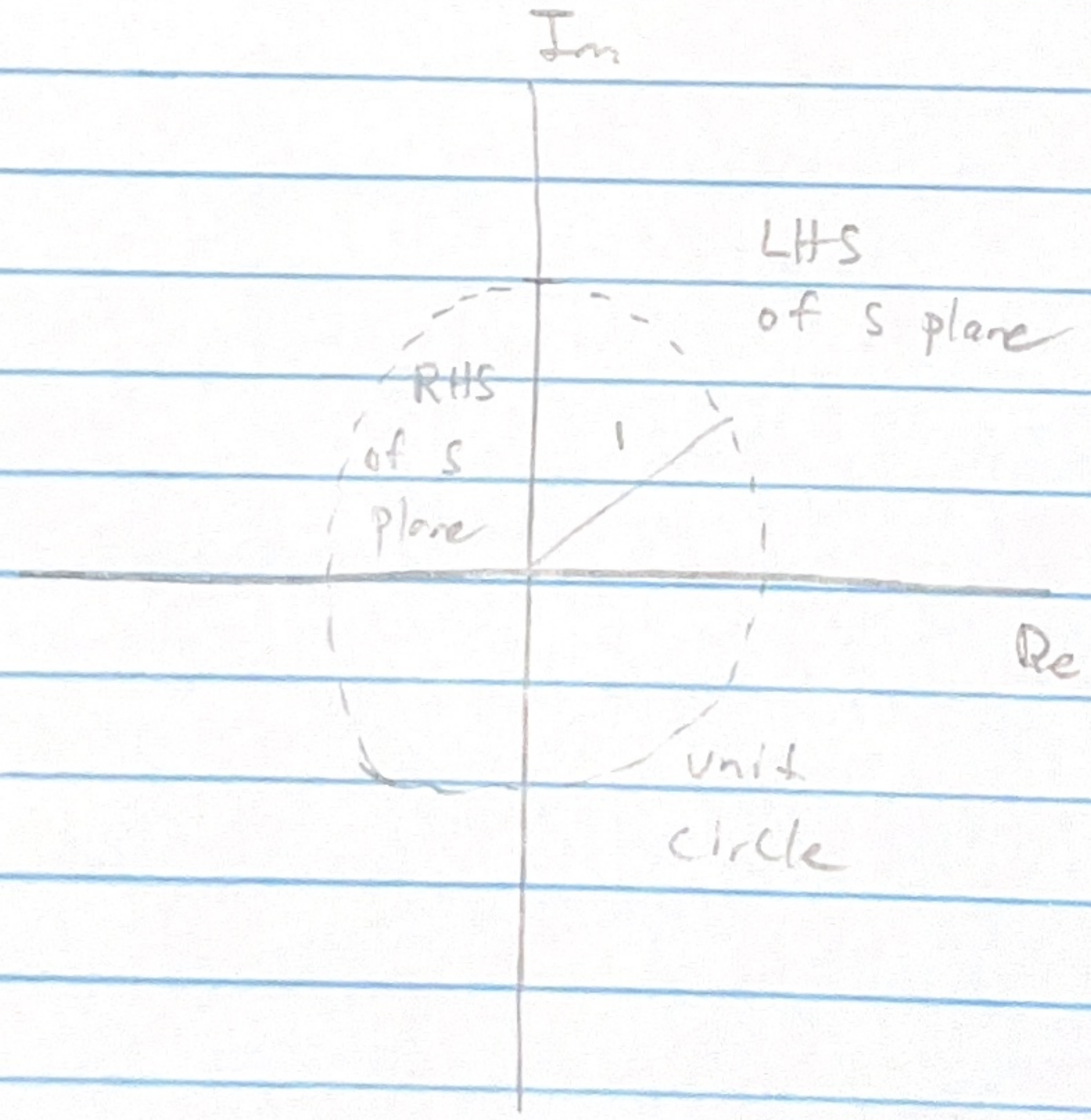
goes to 0 if x[n] is causal

Sampling Process



$$Z[x_s[n]] = L[x_\delta(t)]\Big|_{z=e^{sT_s}} = \sum_n x(nT_s)\,z^{-n}$$

Z - Plane



State Space

SS 11.4

$$x(t) = \begin{bmatrix} V_c(t) \\ i(t) \end{bmatrix}$$

KVL: $V_S = V_L + V_r + V_C = L\frac{di}{dt} + Ri + V_c$

$i = C\dot{V}_c$

$$\dot{x}\begin{bmatrix} i/c \\ \frac{V_S - Ri - V_L}{L} \end{bmatrix} = \begin{bmatrix} \dot{V}_c \\ i\,(\text{dot}) \end{bmatrix}$$

$$\dot{x} = \overbrace{\begin{bmatrix} 0 & 1/2 \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}}^{A} x + \overbrace{\begin{bmatrix} 0 \\ 1/L \end{bmatrix}}^{B} V_s$$

A, B, C, D matrices!

$\dot{x}(t) = Ax(t) + Bu(t)$

$y(t) = Cx(t) + Du(t)$

SS 11.6

$$(sI - A)^{-1} \xrightarrow{L^{-1}} e^{At}$$

matrix exponential

$$\dot{x} = Ax + Bu \xrightarrow{L} X(s) = (sI - A)^{-1}x_0 + (sI - A)^{-1}BU(s)$$

SS 11.7

(A,B) is controllable $\longleftrightarrow$ rank$([B, AB \,\ldots\ldots\, A^{N-1}B]) = N$

rank = number of linearly independent rows/columns

State Space

Example

Transpose $\rightarrow$ $[a_{ij}]^T = [a_{ji}]$

"For RLC Series Circuit"

if $s = \begin{bmatrix} V_c \\ i \end{bmatrix} \rightarrow \dot{s} = \begin{bmatrix} 0 & \frac{1}{C} \\ \frac{-1}{L} & \frac{-R}{L} \end{bmatrix} s + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_s$

let $R = 2$ $L = 1$ $C = 0.5$

$A = \begin{bmatrix} 0 & 2 \\ -1 & -2 \end{bmatrix}$ → Find eigenvectors / Values

$\det(A - \lambda I) = 0$

$v_1 = \begin{bmatrix} 1+j \\ -1 \end{bmatrix}$

Transformation matrix

$T = [V_1 | V_2] \rightarrow T^{-1}AT = D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

$A = TDT^{-1}$

$$\left.\begin{array}{l}\lambda_1 = -1 \\ \lambda_2 = 5\end{array}\right\} \quad \begin{array}{l}\lambda_1^n = \alpha_0 + \alpha_1\lambda_1 \\ \lambda_2^n = \alpha_0 + \alpha_1\lambda_2\end{array} \rightarrow \begin{bmatrix}(-1)^n \\ (5)^n\end{bmatrix} = \begin{array}{c}\begin{array}{cc}(\lambda_i)^0 & (\lambda_i)^1 \\ \downarrow & \downarrow\end{array} \\ \begin{bmatrix}1 & -1 \\ 1 & 5\end{bmatrix}\end{array} \begin{bmatrix}\alpha_0 \\ \alpha_1\end{bmatrix}$$

$$\begin{bmatrix}\alpha_0 \\ \alpha_1\end{bmatrix} = \frac{1}{6}\begin{bmatrix}5 & 1 \\ -1 & 1\end{bmatrix}\begin{bmatrix}(-1)^n \\ (5)^n\end{bmatrix} = \frac{1}{6}\begin{bmatrix}5^n + 5(-1)^n \\ 5^n - (-1)^n\end{bmatrix}$$

$$A^n = \frac{1}{6}[[(5^n + 5(-1)^n] + (5^n - (-1)^n)A] \quad \boxed{DT}$$

$$e^{At} = \frac{1}{6}[(e^{5t} + 5e^{-t})I + (e^{5t} - e^{-t})A] \quad \boxed{CT}$$

$\boxed{\text{Q 1a.)}}$

$X(z) = 2 - \frac{1}{2}z^{-2} + z^{-3}$

$ROC(x) = \{z \neq 0\}$

$\boxed{\text{Q 1b.)}}$ Freq = 2000π rad/s = 1000hz

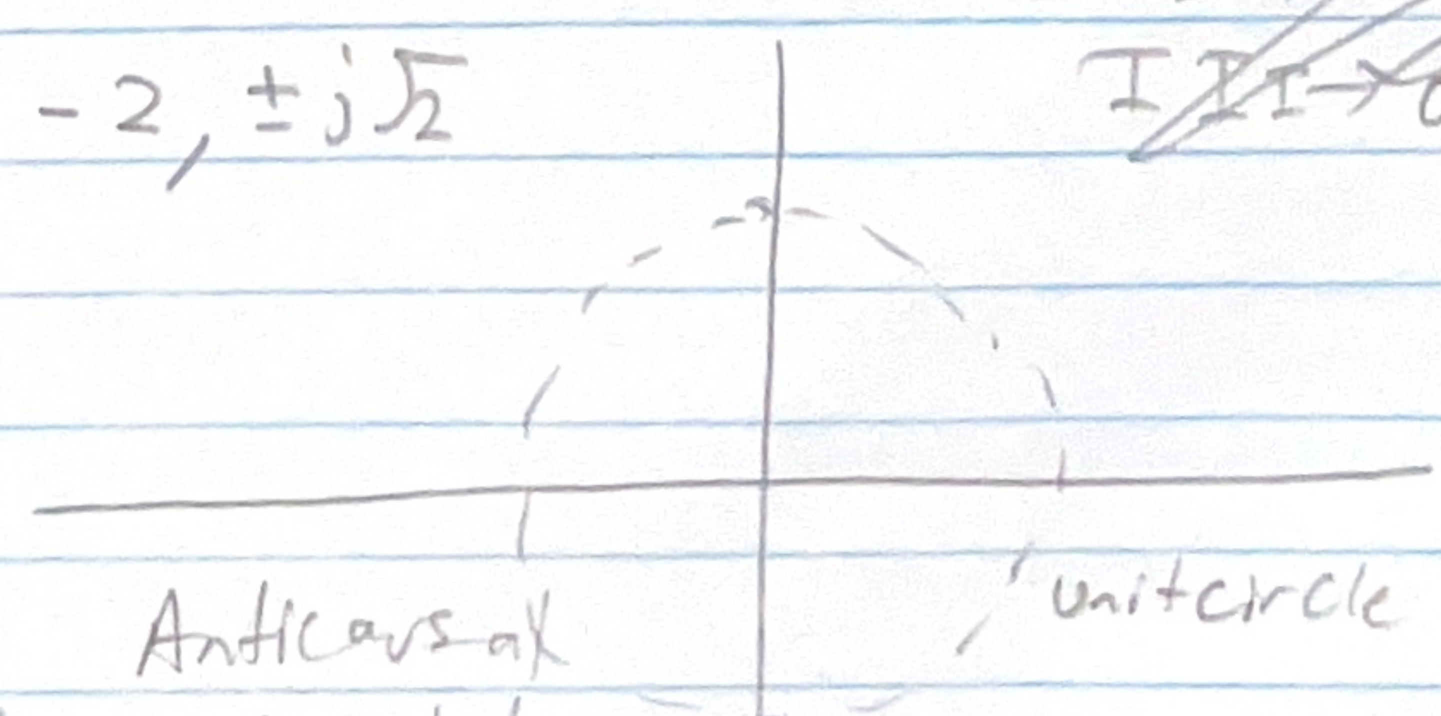
Signal vanishes when $\phi_1 - \phi_2 = \pi$

- Amplitude varies from $2A \to 0$

$\boxed{\text{Q 1c.)}}$

~~I → $-\infty < z < -2$~~
~~II → $-2 < z < 0.5$~~
~~III → $0.5 < z < \infty$~~

$z \neq 0.5, -2, \pm j\sqrt{2}$



Unstable	$0 < \lvert z \rvert < \frac{1}{2}$	Anticausal
Stable	$\frac{1}{2} < \lvert z \rvert < \sqrt{2}$	Two sided
Unstable	$\sqrt{2} < \lvert z \rvert < 2$	Two sided
Unstable	$2 < \lvert z \rvert < \infty$	Causal